LOGIC AND PHILOSOPHY OF LOGIC IN WITTGENSTEIN

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Abstract

This essay discusses Wittgenstein's conception of logic, early and late, and some of the types of logical system that he constructed. The essay shows that the common view according to which Wittgenstein had stopped engaging in logic as a philosophical discipline by the time of writing *Philosophical Investigations* is mistaken. It is argued that, on the contrary, logic continued to figure at the very heart of later Wittgenstein's philosophy; and that Wittgenstein's mature philosophy of logic contains many interesting thoughts which have gone widely unnoticed.

Keywords

Wittgenstein, formal logic, philosophy of logic, Frege, concept-script, language-games

Introduction

> We build ourselves artificial hands—tools for special purposes—which function more exactly than the hand is capable of doing. And how is this exactness possible? Through the very rigidity and inflexibility of the parts, the lack of which makes the hand so dexterous. [Frege 1882: 158]

For early Wittgenstein's philosophy, which culminated in the composition of his *Tractatus Logico-Philosophicus*, formal logic undoubtedly played a central role. The contrary often seems true of later Wittgenstein, in particular of the posthumously published *Philosophical Investigations*. Although, in the preface of the book, logic appears on the list of topics that the book addresses and, arguably, logic indeed receives extensive discussion inside the book, the *Investigations* are not generally considered to have made a direct contribution to logic or the philosophy of logic. Against this common view, I argue that logic played an important role in all of Wittgenstein's work—that is, over the entire period of his philosophising—from both the point of view of the philosopher of logic and that of the logician.

Only a small number of writers have seriously addressed the question what significance logic had for Wittgenstein's philosophy, including for his later philosophy. Amongst these writers, the following remarks with which Ilham Dilman introduces his paper 'Wittgenstein, Philosophy and Logic' [1970] seem to be commonly agreed upon. 'Wittgenstein's chief interest was always in logic and in mathematics, and he considered the study of logic to be fundamental

to philosophy', Dilman writes. 'His discussions in *Philosophical Investigations*, for instance, always gravitate towards questions in the philosophy of logic. Unless one sees how much these are at the centre of his interest one will miss much of what he says on other issues' [ibid.: 33]. So far, this seems right to me. Indeed, as I have already said, I believe that this general point about the importance that the study of logic still had for the later philosophy of Wittgenstein has not received the attention by subsequent philosophers that it deserves. However, it is symptomatic of existing scholarship on Wittgenstein's philosophy of logic that Dilman also considers it necessary to add the following disclaimer: 'By a "study of logic"', he writes, 'I do not mean a study concerned with casting propositions into symbolic form, mapping out their implications, formalizing arguments, developing methods for detecting fallacies in them and checking their validity, discovering logical proofs, constructing formal systems. On the contrary, Wittgenstein repeatedly warned against the tendency towards formalization in philosophy' [ibid.]. This seems wrong to me or, at any rate, misleading.

Of course, Wittgenstein thought that some tendencies towards formalisation in philosophy could be damaging (and who would not think that?), but this does not mean that he thought that all formalisation, or tendencies towards formalisation, in philosophy were problematic. In fact, as I shall try to demonstrate, most of the elements of a study of logic that Dilman thinks later Wittgenstein rejected, except perhaps for the discovering of logical proofs, never ceased to be at the centre of Wittgenstein's interest.

Similarly, at the end of her otherwise admirably comprehensive chapter on 'Wittgenstein on Philosophy of Logic and Mathematics' in the *Oxford Handbook of Philosophy of Mathematics and Logic*, Juliet Floyd makes special note of the fact that 'Wittgenstein's philosophy has helped to inspire the construction of new logics' [2005: 113], such as those that have been developed by Hintikka, Shapiro, Priest and others; but she does not make any mention of later Wittgenstein's own constructive work in logic. Perhaps the most recent example of this tendency in the literature is Penelope Maddy's *The Logical Must: Wittgenstein on Logic* [2015], which I briefly discuss below. The present essay seeks to highlight several of Wittgenstein's, and especially the later Wittgenstein's, constructive contributions to the study of logic.

I shall begin here, in the first of three sections, with some basic distinctions concerning the roles logic plays in Wittgenstein's work, which will be useful for understanding the more intricate details of their respective traits that I shall address in the second and third sections of this essay. These latter sections address the early and the later work of Wittgenstein respectively, with special attention to the *Tractatus* and the *Investigations* as their authoritative expressions.

1. 'Logic'

In one of his final manuscripts, Wittgenstein asks himself the following question:

> Could we imagine a man who keeps on making mistakes where we regard a mistake as ruled out, and in fact never encounter one?
> E.g. he says he lives in such and such a place, is so and so old, comes from such and such a city, and he speaks with the same certainty (giving all the tokens of it) as I do, but he is wrong.

> But what is his relation to this error? What am I to suppose? [MS 174: 15–15v][1]

Then Wittgenstein notes somewhat abruptly:

> The question is: what is the logician to say here? [ibid.]

It is clear from these remarks and others that Wittgenstein, nearing the end of his philosophical development, still sees himself as in an important sense concerned with logic.

The following early remark, from the *Tractatus* [1922], gives expression to one of Wittgenstein's most fundamental convictions concerning the relation between logic and philosophy:

> The object of philosophy is the logical clarification of thoughts. Philosophy is not a body of doctrine, but an activity.
> A philosophical work consists essentially of elucidations.
> Philosophy does not result in 'philosophical propositions', but in the becoming-clear [*Klarwerden*] of propositions. [TLP 4.112][2]

For example, I might be inclined to say, 'some systems of formal logic describe the logic of language'. According to Wittgenstein's conception of philosophy, it would be wrong to think that uttering such a sentence would, as such, be the result of philosophy; 'philosophical propositions', as it were. Rather, my utterance of this sentence marks the beginning of the real philosophical work that is needed. Wittgenstein later wrote in the *Investigations*: 'What we are "tempted to say" in such a case is, of course, not philosophy; but it is its raw material' [PI §254]. Hence, what the philosopher has to do in such a case is to make clear—to themselves as much as to whoever might care to find out—what exactly might be meant by what they were tempted to say. This activity is what Wittgenstein calls the logical clarification of thoughts.

It is instructive to note just how much of this logico-philosophical work is required in order, for instance, to make clear the meaning of the remark that I just quoted from the *Tractatus* (see indentation above). The following story aptly illustrates this. Upon having read the manuscript of the book 'twice carefully', Russell remarked in a letter to Wittgenstein, '4.112. I agree strongly with this number.'[3] Of course, he would: Russell could have written this remark himself. In fact, however, the two deeply disagreed about this particular remark as well as about logic and the philosophy of logic in general.

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[1] This manuscript passage, including the following quotation, was translated in OC [§§67–8]. Works by or originating from Wittgenstein are cited using abbreviations following standard practice; the list of references contains the key to all abbreviations.

[2] Translations of the *Tractatus* are taken from the Pears/McGuinness translation or the Ogden/Ramsey one or both, without further indication. Translations have been emended where necessary, also without further indication. The same applies to the Bartlett translation of Frege's 'Über die wissenschaftliche Berechtigung einer Begriffsschrift', as quoted in the epigraph to this essay. Translations of Wittgenstein's *Investigations* are taken from the Hacker/Schulte translation; in this case, solely the translation of section 81 has been emended (namely to read 'as if what we are [not "were"] talking about in logic…', because the counterfactual of the original '*redeten*' is expressed by the subsequent 'were' in '…were an *ideal* language').

[3] The letter is dated 13 August 1919.

1.1 Two uses of 'logic'

Wittgenstein mainly uses the term 'logic' in two ways: either to refer to a system of formal logic such as Aristotle's logic, Frege's concept-script or Russell's *Principia Mathematica* (hence, to refer to the method or tool of investigation), or to refer to how language in general, one language in particular, or specific expressions, propositions, words, concepts, etc. function (hence, to refer to the object of investigation).[4]

Many important issues in the philosophy of logic can be framed with regard to possible relations between logic as the method or tool of investigation and logic as the object of investigation. In what sense are systems of formal logic *about* the logic of language? Is there only one correct system of formal logic or are there many? Here is a relatively late remark, from 1948, in which Wittgenstein discusses a related kind of issue:

> Aristotelian logic brands a contradiction as a non-sentence, which is to be excluded from language. But this logic only deals with a very small part of the logic of our language. (It is as if the first geometrical system had been a trigonometry; and as if we now believed that trigonometry is the real basis for geometry, if not the whole of geometry.) [MS 137: 129–129v][5]

According to Wittgenstein, there exists a general tendency to reify systems of logic or, what comes to the same, 'to sublimate the logic of our language' [PI §38]. In other words, what is at issue in the above passage is a tendency to misunderstand what might be called the normative character of formal logic in such a way, for instance, as to try to eliminate features of our ordinary language that do not accord with the rules of some system of formal logic even though these features of our ordinary language might in fact fulfil genuine functions that merely cannot be captured adequately by this particular set of rules. Against such reifying and sublimating attitudes, Wittgenstein reminds us that sometimes expressions such as 'Yes and no' fulfil a vital communicative function in virtue of being contradictory, for example in replying to questions such as 'Do you love me?' or 'Are you a Marxist?'.[6]

In the remainder of the present section, I shall briefly say a few things about the development of Wittgenstein's general concern with the logic of language. In the subsequent two sections, I shall then attempt to illustrate some of the roles that formal logic played in Wittgenstein's related philosophical efforts.

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[4] Regarding my own use of the term 'formal logic' in this essay: I do not mean to impose any particular notion of formality. Rather, I am using the term here mainly to indicate, where necessary, that I am speaking of logic as a method or tool of investigation and in contradistinction to what is commonly called informal logic; I take the latter distinction to be simply that informal logic is so called because it involves a lesser degree of formalisation than what is called formal logic.

[5] This manuscript passage was translated in LW I [§525].

[6] Cp. PI [§81]; see Railton [2000] for detailed discussion of this section.

1.2 The logic of language

The following passage from the Preface of the *Tractatus* contains one of the most salient instances in Wittgenstein's work of 'logic' being used to refer to the object of investigation. He writes:

> The book deals with the problems of philosophy, and shows – as I believe – that the posing [*Fragestellung*] of these problems stems from misunderstanding the logic of our language.

Wittgenstein thinks that there is a characteristic dimension to philosophy which is manifested in a powerful, and dangerous, tendency to talk nonsense without being aware of it.

In his later writings, Wittgenstein speaks less frequently of the 'logic' of our language. Instead, he now often speaks of its 'grammar'.[7] More frequently still, later Wittgenstein speaks of the grammar of particular words or expressions.[8] I do not know what prompted later Wittgenstein to start referring to the object of his investigations as 'grammar' rather than as 'logic'. For present purposes, it will be safe to assume that this terminological shift makes no significant difference. Several commentators have pointed out, correctly in my view, that it is no easy task to say what later Wittgenstein's conception of grammar is, given how variously he uses the term in characterising what he is concerned with; for instance, he also speaks of 'grammatical illusions', 'grammatical fictions' and 'the rules of grammar' (see PI [§§110, 307, and 497]; cp. McGinn [2011]). However, the same is arguably true of Wittgenstein's conception of logic; while particular instances of either expression in Wittgenstein's work tend to be unproblematic.

For example, in many passages in which Wittgenstein speaks of grammar he is referring to what might also be called 'surface grammar', in others he is referring to what might be called 'depth grammar' (cp. PI [§664]). In the *Tractatus*, Wittgenstein describes the same difference in terms of logic: 'Everyday language is a part of the human organism and is no less complicated than it. It is not humanly possible to gather immediately from it what the logic of language is' [4.002]. In a late remark, again, he puts it as follows: 'The logic of language is immeasurably more complicated than it looks' [MS 169: 72v].[9]

The shift towards greater attention to the specific features of particular words and expressions in the later period corresponds to, amongst other things, Wittgenstein's growing appreciation of just how difficult a task it is to describe the logic of language at all clearly, even in what appear to be the simplest cases. Throughout this transition, Wittgenstein remains

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[7] In *Philosophical Investigations*, the only two instances of the 'of our language'-variety of 'logic' occur in sections 38 and 93. The corresponding use of 'grammar' occurs, for example, in sections 29, 122, 295, 354, 371, 373, 497, 520, and 528.

[8] For instances of 'grammar' of particular words or expressions, see PI [§§35, 150, 165, 182, 187, 199, 257, 339, 492, 657, 660, 664, and 693]. For a corresponding use of 'logic', see for instance PI [§345].

[9] The first sentence of this remark, preceding the one quoted in the main text, reads 'Bad influence of Aristotelian logic.' The whole passage was translated in LW II [44].

committed to the same basic principle, however: if we want to understand the logic of language, we have to look at how language is actually used in life. He writes:

> In philosophy the question, 'What do we actually use this word, this sentence for?' leads to valuable insights, time and time again. [TLP 6.211][10]

In the course of his steadily growing awareness of the difficulties one faces in trying to answer this kind of question and, as a consequence, his increasing focus on the situational particularity of language in use, Wittgenstein undertook a substantial elaboration of the formal logic that he employed.

2. Concept-scripts and diagrams

In 1933, Wittgenstein had a large collection of philosophical remarks typed up. The typescript comprises a total of 768 pages and includes many handwritten revisions which Wittgenstein continued to work on until at least 1937. The resulting text has been published posthumously and is now widely referred to as Wittgenstein's *Big Typescript*. The text is divided into chapters and subchapters. One chapter is titled 'Grammar'; and the most extensive subchapter of this chapter deals primarily with questions concerning logical analysis and the nature of formal logic.[11] At the heart of this subchapter is a discussion which Wittgenstein eventually incorporated into section 81 of the *Investigations*, where he warns us that

> it may look as if what we are talking about in logic were an *ideal* language. As if our logic were, so to speak, a logic for a vacuum. – Whereas…the most that can be said is that we *construct* ideal languages. But here the word "ideal" is liable to mislead, for it sounds as if these languages were better, more perfect, than our everyday language; and as if it took a logician to show people at last what a proper sentence looks like.

We do not need a logician to show us what a proper sentence looks like, of course. Nor do we usually need one to tell us what we mean by our words. We do not usually need a logician either to point out to us things such as that, strictly speaking, certain standard phrases at the beginning of a book involve a contradiction (the preface paradox). On the other hand, we all need a Socrates, at least every now and then; someone, that is, to help us make sense of things when we are at a loss. In turn, every Socrates loves a Frege—the builder of tools for sense-making—and so did Wittgenstein.

Frege had indeed constructed his concept-script (*Begriffsschrift*) not as some kind of ideal that ordinary language would have to be brought in line with but in response to, as he writes, 'the lack of a device to avoid misunderstanding in others as well as errors in one's own thinking [that] makes itself so often felt in the more abstract scientific disciplines' [1882: 155]. 'May philosophers

[10] On the development of Wittgenstein's attitude towards ordinary language, see Conant [manuscript].

[11] The subchapter extends over fifteen pages of typewriting and includes many handwritten revisions. The title is as follows: '58. The Strict Grammatical Rules of a Game and the Fluctuating Use of Language. Logic as Normative. To What Extent Do We Talk about Ideal Cases, an Ideal Language? ("The Logic of a Vacuum.")'.

too, then, give some attention to the matter!' he added [ibid.: 160]. That is what Wittgenstein did (cp. Diamond [1984]).

I now want to briefly discuss a small selection of insightful remarks about logic which can be found in the *Tractatus*, before turning to the later logic. Of course, at the end of that book Wittgenstein tells us: 'My sentences [*Sätze*] serve as elucidations in the following way: whoever understands me, finally recognises them as nonsensical' [6.54]; but even if Wittgenstein's *Sätze* are thus nonsensical, we may still be able to single out and cultivate some healthy-looking sprouts.[12]

2.1 Diagrammatic notation in the Tractatus

The signs, strings of signs or structures of signs in formal logic, which we manipulate according to the rules of the system, are significantly different from the sentences of our ordinary language. One essential difference is as follows. If one wanted to conceive of formal strings of signs as something like logical sentences, which could be true or false, then on closer inspection one would find that they can fulfil their function equally well when construed in the form of tautologies; that is, when construed in such a way that they cannot be false.[13] In the *Tractatus*, Wittgenstein writes:

> 6.12 … If propositions are to yield a tautology when they are combined in a certain way, they must have certain structural properties. So their yielding a tautology when combined *in this way* shows that they possess these structural properties.
>
> …
>
> 6.121 The sentences of logic demonstrate the logical properties of propositions by combining them so as to form sentences that say nothing.

The fact that so-called logical sentences can, apparently, not be false has led many to think that therefore they must be true, hence, that they are necessary truths; but, alternatively, Wittgenstein's point can make it seem doubtful whether one should use the term 'sentences' here at all. If so-called logical sentences cannot be false, then can they be true? Perhaps it would be wiser to avoid this analogy between language and systems of formal logic. Wittgenstein continues:

> It follows from this that we can actually do without logical sentences; for in a suitable notation we can in fact recognize the formal properties of propositions by mere inspection. [TLP 6.122]

The passage ends with an example of such a notation: 'In cases where no generality-sign occurs', Wittgenstein explains, 'one can employ the following illustrative method.' Namely, instead of writing ~(p.~q) and its truth table, for instance, we can draw the following diagram:

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[12] Cp. Kremer [2001]. For useful presentations of some healthy-looking sprouts, and of some of the dead wood, see Glock [1996].

[13] Russell expressed his agreement with this point in a letter to Wittgenstein dated 13 August 1919. In addition to the passages that I have quoted in the main text, see also TLP [4.461, 6.1–6.111, and 6.1221].

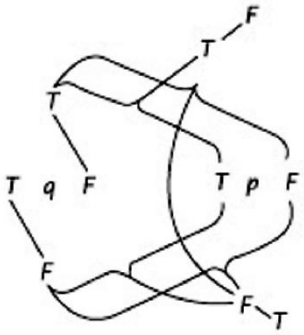

Figure 1: Diagrammatic notation [TLP 6.1203]

Let us suppose that this diagram represents a correct truth-functional analysis of what someone meant when they uttered the following words: 'It is not true that Wittgenstein threatened and Popper did not provoke.'

The diagram lets us see that what the person meant to say would be true even if it were actually true that Wittgenstein threatened, provided that it were also true that Popper provoked.[14] So, perhaps, what the person meant to say could have been expressed more clearly by saying 'Wittgenstein might have threatened, but Popper definitely provoked.'

Alternatively, using the notation of *Principia Mathematica* (PM), we can say that the person who uttered these words did not mean something of the form ~p.~q, for which their utterance could easily be mistaken. However, unlike PM-notation, the diagram illustrates the truth-functional structure of what was said without making use of anything that one might even be tempted to call 'logical sentences'.

In one sense, Wittgenstein's diagrammatic notation simply follows Frege's guidelines for (two-dimensional) logical notations. Frege writes:

> [A well-constructed logical notation] will have to be entirely different from all word-languages in order to make full use of the specific advantages of visual signs. … Such brevity must thereby be striven for that the two-dimensionality of the writing surface can be well exploited for the perspicuity of the representation. [1882: 159–60][15]

Furthermore, the logician's construction of suitable notations can itself be regarded as an act of philosophical clarification. As Wittgenstein writes, 'we have the right logical point of view once all is in order in our sign-language' [TLP 4.1213]. After all, it is in virtue of the skill of the logician as a philosopher that the particular act of logical clarification can succeed without at the same time provoking a number of puzzling questions which would bring itself into question; for instance, questions concerning the semantic or epistemic status of notational features. Are tautologies of classical logic, such as that of non-contradiction ~(p.~p), necessary truths about the world? Do elementary propositions (or atomic facts, or possible worlds, etc.) exist?

[14] The corresponding reading of the lines of the diagram starts from the bottom and sees the following connections: *T – T p – T q*.

[15] In this respect, Frege's notation is unrivalled by that of *Principia Mathematica*.

Thus, the diagram fulfils its function not only without appearing to formulate something that might be called 'logical sentences' but also without making use of logical constants—such as, in PM-notation, v, ⊃, ~, ≡ and .—whose nature has been the subject of endless controversies amongst philosophers of logic; all that there is in the diagram is *the line* connecting various *T*s and *F*s.[16] For example, the conditional of classical logic (⊃ in PM-notation), as it would feature in the simplified classical analysis of our example above following the equivalence of ~(p.~q) and p⊃q, has been criticised for just this kind of formalisation because, unlike our example above, ordinary language conditionals (if…then) are often false in case the antecedent is false and the consequent is true; but the line cannot be so criticised, because it clearly does not involve any such general claim about the semantics of ordinary language conditionals.

The notational minimalism of the line goes beyond that of the Sheffer stroke and Wittgenstein's own N-operator (cp. TLP [5.131, 5.502, 5.51, and 6]). In 5.101, Wittgenstein introduces yet another notational device according to which our above example would be formalised as (T T F T) (p,q) with the first set of parentheses giving the truth-functional values for possible truth-value combinations of p and q. This notation may indeed match the line's minimalism, in virtue of making no apparent use of logical constants either. However, it lacks the perspicuity of the diagrammatic notation: it can hardly be said to let us 'recognize the formal properties of propositions by mere inspection' [6.122].[17]

Perhaps the full truth-table notation in 4.31 and 4.442, from which the 5.101 notation is derived, matches the diagrammatic notation in terms of perspicuity so defined. Compare:

p	q	not q	p and not q	it is not the case that p and not q
T	T	F	F	T
F	T	F	F	T
T	F	T	T	F
F	F	T	F	T

Figure 2: Full truth-table notation

However, the fact that what I have called Wittgenstein's diagrammatic notation manages with exactly half the number of *T*s and *F*s shows, I think, that out of all notations that the *Tractatus* offers the diagrammatic notation is ultimately the most successful in following Frege's instruction to strive for 'such brevity…that the two-dimensionality of the writing surface can be well exploited for the perspicuity of the representation'.

2.2 An objection

Someone might object as follows: 'This diagrammatic notation is nothing but a crude equivalent to more elaborate logics. In particular, it fails to make explicit the system of rules according to

[16] See also TLP [5.4]: 'Here it becomes manifest that there are no "logical objects" or "logical constants" (in Frege's and Russell's sense).' See further 5.441, 5.53, and 5.531–5.5321, and the related proposal of a solution to Russell's Paradox in 3.333.

[17] I am grateful to an anonymous referee for inspiring the comparison with the 5.101 notation.

which it operates. *Principia Mathematica*, for instance, does a much better job at formulating these "primitive propositions" of logic.'

Perhaps the diagrammatic notation cannot be fully developed in terms of formal semantics or as an axiomatic system. If it cannot be so developed, this will raise some interesting questions. However, the objection strikes me as an unfounded call to construct another logical system in order to model one we already have. For, one might ask, to whose benefit should this additional system be constructed? To the extent that the diagram sufficiently clarifies the relevant thought, the job of the logician has been done. Not every logician and philosopher has to be a meta-logician in the way the objection seems to suggest. Questions of metalogic—for example whether a given logical system is complete in the sense that all of its tautologies are provable in some deductive system—are no doubt important, not least to mathematical logic and computer science. However, the purpose of a logical system may be quite independent of its metalogical properties; a logic that is incomplete in the mentioned sense is not therefore a useless logic, for instance. In particular, the incorporation of metalogical results—such as 'primitive propositions' or the adequacy of v and ~ in PM—is by no means imperative, which is presumably why standard classical logic has in practice ignored such adequacy results. Therefore, the diagrammatic notation might be a good logic for philosophy even if it cannot be fully developed in terms of formal semantics or if it cannot be axiomatised.

In this section, I have argued that Wittgenstein's diagrammatic notation in the *Tractatus* constitutes a system of formal logic that is supposed to help clarify problematic propositions while keeping philosophical contention to a minimum. I have further argued that constructing logical notations is thus itself an act of clarification; for instance, constructing a logical notation may help us avoid mistakes about, and lead us to an improved understanding of, the nature of what is, or can be, logically constant.

3. Language-games as logic

If constructing logical notations is itself an act of clarification—in that logical notation is supposed to help clarify problematic propositions while keeping philosophical contention, or the appearance of it, to a minimum—then, the logician or philosopher may ask, how might logical problems be solved without thereby engendering new problems? Arguably, later Wittgenstein's language-games are designed to do just that. Thus, I argue, they constitute a logic for philosophy 'so that it is no longer tormented by questions which bring *itself* in question' [PI §133].

Since 'the very nature of the investigation…compels us to travel criss-cross in every direction over a wide field of thought', Wittgenstein writes in the Preface to the *Investigations*, 'this book is really just an album.' There is, therefore, more than one way of reading the text: there are, as it were, different ways of browsing the album. One way of reading it sees Wittgenstein's language-games as constituting a new philosophical logic.[18]

[18] Williamson [forthcoming] gestures towards this kind of reading. Oskari Kuusela has recently presented an account of language-games that is similar to the one I develop here: Kuusela [2014] cites many supporting passages from Wittgenstein's manuscripts and offers helpful comparisons with Frege and Russell; see also Kuusela [manuscript]. Cp. also Travis [2006: ch. 3].

This reading is perhaps currently not likely to strike many readers as the most intuitive one. For example, while Maddy in her recent book-length study of Wittgenstein's philosophy of logic agrees that there is not just one useful way of reading the *Investigations*, it is striking that her 'readings that focus on logic' [2015: 4, n. 6] make no mention whatsoever of any constructive element in Wittgenstein's later writings on logic and, further, that she thinks that 'discussion of logic in the *Philosophical Investigations* is confined almost exclusively to the meta-philosophical passages §§89–133' [ibid.: 74]. In fact, however, the text contains a variety of more or less explicit indications that point towards a reading according to which discussion of logic is indeed pervasive in Wittgenstein's *Investigations*. Some of what are perhaps the most important elements of this discussion can be found in the following selection of passages:

> 5. … It disperses the fog if we study the phenomena of language in primitive kinds of use in which one can clearly survey the purpose and functioning of the words. …
>
> 7. … I will call these games "*language-games*" and will sometimes speak of a primitive language as a language-game. … I shall also call the whole, consisting of language and the activities into which it is woven, a "language-game".
>
> 81. … in philosophy we often *compare* the use of words with games, calculi with fixed rules, but cannot say that someone who is using language *must* be playing such a game. — But if someone says that our languages only *approximate* to such calculi, he is standing on the very brink of a misunderstanding. For then it may look as if what we are talking about in logic were an *ideal* language. As if our logic were, so to speak, a logic for a vacuum. – Whereas…the most that can be said is that we *construct* ideal languages. …
>
> 130. Our clear and simple language-games are not preliminary studies for a future regimentation of language – as it were, first approximations, ignoring friction and air resistance. Rather, the language-games stand there as *objects of comparison* which, through similarities and dissimilarities, are meant to throw light on features of our language.

We can identify the following three features as central to the conception of language-games as logic that thus emerges.[19]

First, language-games are *clear and simple*. They are clear because they are relatively simple, or 'primitive' [§5], as compared with the language, or aspects thereof, that they are being used to analyse; they thus tend to resemble, as Wittgenstein puts it, 'those games by means of which children learn their native language' [§7].[20]

Second, language-games are *holistic*. A language-game is, as Wittgenstein writes, 'the whole, consisting of language and the activities into which it is woven' [§7]; or, as he also puts it, 'to imagine a language means to imagine a form of life' [§19]. Note also that language-games

[19] It should be noted that Wittgenstein uses the term 'language-game', like 'logic' and 'grammar', not only to speak of a technical instrument of logical analysis but also to refer to our language as a whole or certain parts of it. In this alternative sense, the term mainly stresses the interwovenness of language with life and, in particular, action. Insights into this interwovenness guided Wittgenstein's development both as a philosopher and as a logician.

[20] On Wittgenstein's uses of the word 'primitive' in the opening sections of the *Investigations*, see Schulte [2004: 23–5].

are similar to what are called ‘possible worlds’ in modal logic in this latter respect. Like a possible world, a language-game stipulates a model that is complete. Therefore, given that the stipulation is consistent, questions as to how the model is possible will be misplaced: the model (possible world, language-game), once introduced, is what it is. I take this to be implied by Wittgenstein’s asking us to conceive of the builders language-game in section 2 ‘as a complete primitive language’.[21]

Thirdly, language-games function as *objects of comparison*. A language-game might be so constructed that it displays relevant qualities that differ significantly from those of the language, or aspects thereof, that they are being used to analyse; in many cases, it will be obvious that a given language-game is not supposed to be anything like a formal equivalent or abstraction. This makes formalisation using language-games a somewhat more difficult task than it tends to be with most other logics. Consequently, the comparison of a given formalisation with the object under investigation plays a somewhat more important role than it tends to do in most other logics or in the way they are standardly applied.

Wittgenstein’s most famous language-games include, of course, *the shopkeeper* and *the builders*. However, there are many more in the *Investigations*: for instance, the language-game of *writing series of numbers* in section 143, a variation of which—*the recalcitrant student* in section 185—gives rise to Wittgenstein’s famous discussion of rule-following; and that of *private sensations* in section 243, the subsequent discussions and variations of which are among the most influential parts of Wittgenstein’s work.

We may now also see the point of Wittgenstein’s memo to himself: ‘The question is: what is the logician to say here?’. For in the notebook passage where it occurs, as quoted at the start of the first section of this essay, Wittgenstein is again in the business of constructing language-games.

3.1 An example

The most elaborately developed example of a language-game in the *Investigations* is that of the builders. Wittgenstein first introduces the language-game of the builders in section 2. He goes on to construct six major variations of this language-game—in sections 8, 15, 21, 41, 42 and 86—and many of the surrounding sections are devoted to the application of these language-games; in particular, all of the first thirty or so sections.

In order to distinguish the original and its variants, Wittgenstein also refers to them by section numbers. Here, then, is (2), the original builders:

> *A* is building with building stones: there are blocks, pillars, slabs and beams. *B* has to pass him the stones and to do so in the order in which *A* needs them. For this purpose

[21] Of course, we can always find or invent new models (possible worlds, language-games), including variations in terms of specific forms of life; but it is important to distinguish sharply between using a given model and finding or inventing a new one. The criticism by Rush Rhees, who claims that it is not in fact conceivable that the builders’ language be complete, misses its target because it fails to respect this distinction; cp. Rhees [1960: 177–80].

> they make use of a language consisting of the words “block”, “pillar”, “slab”, “beam”. *A* calls them out; *B* brings the stone which he has learnt to bring at such-and-such a call.

The main purpose of (2) is to clarify the thought that words stand for things (cp. Perry [1994]). More fully expressed, the thought goes something like this: ‘Every word has a meaning. This meaning is correlated with the word. It is the object for which the word stands’ [§1]. The thought is natural enough and it will indeed, I think, seem true to most people. It certainly seems true to me. In fact, I am quite convinced that it is true.

Wittgenstein illustrates the thought, and its intuitiveness, in the form of the notorious quotation of Augustine with which the *Investigations* begin. His subsequent discussion shows that the expression of this thought, for all its intuitiveness, is much more difficult than it seems. It is obviously important that, as reflective human beings, we must not be deceived about the nature of meaning. Anyone who wants to insist that words stand for things will therefore want to know exactly how it may be so. Thus, Wittgenstein is quite right that the proper grasp of this thought deserves a good deal of logical clarification.

The fact that the builders serve to analyse a thought about meaning—specifically, that they serve to answer the question of what some expression about meaning means—has made it relatively difficult for readers to see clearly what exactly Wittgenstein is doing; while at the same time, of course, this structure makes what Wittgenstein is doing rather exciting, at least from the point of view of a logician. In order to avoid confusion, I shall henceforth use ‘*r*’ to abbreviate the expression ‘words stand for things’.

Seeing how (2) satisfies the three features that I have said are central to the conception of language-games as logic will help us see how the logical analysis of *r* succeeds.

First, it is *clear and simple*. For instance, unlike English or Chinese, (2) contains only four words; and, unlike the real world, (2) contains only a very small number of objects, namely four different kinds of stone.

Second, it is *holistic*. Besides words and things, (2) contains activities that are intimately connected with its linguistic repertoire.

Thirdly, it functions as an *object of comparison*. Wittgenstein announces (2) by saying: ‘Let us imagine a language for which the description given by Augustine is right’; hence a language for which *r* will be a true statement. So (2) is not modelled after English or, for that matter, any other natural language; it is not supposed to be a formal equivalent or abstraction. Rather, (2) is a formalisation in the shape of a model of a language that, like some remotely possible world, may be entirely different from our own and all that we have previously known. Hence, it is essential for the logical analysis of *r* via language-game (2) that the analysis will partly consist in comparing (2) with whatever language it is being used to analyse. Of course, any such comparing may require us to add further objects of comparison and hence to construct additional language-games, as illustrated by Wittgenstein’s own procedure.

3.2 What is essential to a language-game

Someone might object to the foregoing discussion: 'If Wittgenstein intended language-games to form anything like a system of formal logic, then why would he not say so, or at least say so more clearly; and why would he not develop this system at all rigorously?'

Wittgenstein formulates a similar objection to himself in section 65 of the *Investigations*: 'You make things easy for yourself! You talk about all sorts of language-games, but have nowhere said what is essential to a language-game.'

'I'll try to explain this', Wittgenstein says. There then follows the well-known stretch of remarks in which he argues that there is no explanatory set of jointly necessary and sufficient conditions for all of the activities that we call 'games': the many activities that we call 'games' are simply too diverse; in fact, there cannot even be a definite disjunction of all of the features of activities that we call 'games', because there are indefinitely many possible types of activities that we may or may not call 'games'. If this is so, Wittgenstein continues his argument, then explaining to someone what a game is by giving various examples of games—as we normally would—'is not an *indirect* way of explaining – in default of a better one' [§71]; and the same is true of explaining to someone what a language-game is.[22] Hence, as Wittgenstein puts it:

> One gives examples and intends them to be taken in a particular way. – I do not mean by this expression, however, that he is supposed to see in those examples that common feature which I – for some reason – was unable to formulate, but that he is now to employ those examples in a particular way. [§71]

This is, of course, exactly what Wittgenstein does. The term 'language-game' is introduced in section 7 of the *Investigations*. By that stage, Wittgenstein has already made substantial use of several language-games and also given some relevant practical instruction. In fact, section 7 introduces the term 'language-game' by reference to the example of language-game (2). Wittgenstein's explanation of what a language-game is thus mainly amounts to a sort of practical initiation: he shows the reader many examples of language-games, what to do with them and how to construct similar ones; it is not until section 65 that he raises the question regarding the nature of language-games and how to explain it.[23]

Wittgenstein's argument still leaves open several important questions. I believe that he has something useful to say on most of them. Here, however, I can only give a brief sketch. First of all, even if Wittgenstein is correct about games, there will remain the question whether the word 'game' is actually employed in the term 'language-game' in such a way that the argument works for language-games as well as it does for games. I think that the requisite analogy does indeed hold: because, to put it very briefly, there are indefinitely many possible types of linguistic activity that may serve as language-games, which corresponds to an indefinitely large number of possible problems that require solving; hence, as is arguably the case with 'games',

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[22] Having thus reached the conclusion of the argument, Wittgenstein adds a subtle reminder of the intended connection between game and language-game that was first made in sections 65 and 66: 'This, after all, is how we play the game. (I mean the language-game with the word "game".)' [§71].

[23] Wittgenstein's discussion of the argument in sections 65 to 71, which I have rehearsed above, then continues through to section 81 where he turns the discussion of language-games into an explicit discussion of logic which, in turn, continues for over twenty more sections.

the many things that are called ‘language-games’ are so diverse that there can be no explanatory set of jointly necessary and sufficient conditions for all of the things that are so called. Of course, this kind of answer itself raises important questions. For instance, what exactly are the problems, or kinds of logical problems, that may be solved via the construction of language-games? I cannot begin to address these questions here.

Conclusion

Like other formal logics, language-games provide clear and simple models for logical analysis. I have already noted that formalisation using language-games tends to be more difficult than with most other logics, because a language-game is not supposed to be a formal equivalent or abstraction; rather, a language-game is a model of a language that, like some remotely possible world, may be entirely different from what it is being used to analyse; thus the comparison of a given formalisation with what it is being used to analyse plays a somewhat more important role than it tends to do in the way most other logics are standardly applied.

In this latter respect, then, formalisation using language-games is rather like the construction, as opposed to the mere application, of formal logic. Indeed, formalisation using language-games potentially involves the construction of a new system of formal logic; language-game (2), for instance, constitutes a new system of formal logic in this sense.

Existing logics can be used as sources of language-games, too. After all, the question what might make a given statement true remains a leading principle of formalisation; for example: the shopkeeper language-game in section 1 is designed to make the statement ‘Words stand for things’ false, while the language-game of the builders in section 2 is designed to make the statement true. Therefore, any system of formal logic can in principle be used as a source of language-games involving some ‘primitive’ activity like that of Wittgenstein’s builders.

The search for a philosophical logic that solves problems without thereby engendering new ones led Wittgenstein to the conception of language-games as logic.[24] I have said that the construction of formal logic may thus itself become an act of clarification: namely, to the extent that one’s logic is supposed to help clarify problematic propositions while keeping philosophical contention, or the appearance of it, to a minimum. With the conception of language-games as logic, these two acts of logical clarification—the construction of formal logic and its application—become one. Consequently, Wittgenstein’s explanations in *Philosophical Investigations* of what a language-game is do not merely constitute the presentation of a new system, or new systems, of formal logic for philosophy; rather, he is trying to teach the very art of the philosophical logician, as he sees it.

[24] The extent to which Wittgenstein’s early work in logic can be seen as preliminary to his mature work on language-games is an interesting question. I have suggested that some of his early work, such as the diagrammatic notation, betrays a philosophy of logic that is significantly closer to that of the later Wittgenstein than is commonly appreciated. Cp. Sullivan [2003].

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