Paradox

A new theory of paradox is presented, according to which a paradox is something that is beyond belief. Using a large variety of examples, it is demonstrated how this theory avoids the problems associated with previous accounts (including Quine's and Sorensen's); the objection of excessive inclusivity receives a response; and some further analysis is offered of the nature and logic of paradoxical objects and the epistemic value of finding things paradoxical.

Word count: 7,110

> If the paradox-monger is good at his work,
> we stand to learn something.
>
> — David Lewis

A paradox is something that is beyond belief. Indeed, that is the essence of a paradox. If something is a paradox, then it is beyond belief; and if something is beyond belief, then it is a paradox. It is that simple.

Some people may find this hard to believe. To that extent, this account may itself be beyond belief, thus entailing that it too is a paradox. Fortunately, some paradoxes are true.

Of course, even if paradoxes are essentially beyond belief, they need not be beyond *all* belief. Whether or not something is beyond belief usually depends on the subject whose belief or lack of belief is in question.

Two obvious objections to this kind of account are that it might seem overly inclusive and that it might seem overly subjective: more specifically, it might seem to classify too many things—or, perhaps, kinds of things—as paradoxes, and it might seem to classify as paradoxes more or less anything that some one subject fails, for whatever reason, to believe.

In fact, the account is neither too inclusive nor too subjective. To see this, it helps to see first what is wrong with alternative accounts of paradox.

A type of account that has recently enjoyed significant popularity is one according to which a paradox is an inconsistent set of individually plausible propositions.[1] For example, take the paradox of the barber who shaves all and only those who do not shave themselves. According to the set-based account, the barber paradox could perhaps be represented as follows.

[1] See, for example, Rescher 2001, Schiffer 2003 and Lycan 2010.

p_1: There exists a barber who shaves *x* if, and only if, *x* does not shave *x*.

p_2: Either the barber shaves *x* or the barber does not.

p_3: Either the barber shaves the barber or the barber does not.

p_4: If the barber shaves the barber, then the barber does not shave the barber.

p_5: If the barber does not shave the barber, then the barber shaves the barber.

Complications arise, for any set-based account, from the apparent variety of sets of propositions that may reasonably be taken to represent the same paradox. For instance, it may be argued that p_2 and p_3 above are unnecessary; or, indeed, that p_1 alone would be sufficient in virtue of being both (apparently) plausible and (arguably) inconsistent. How should it be decided which, if any, of these alternative sets is, or is not, the relevant paradox? Thinking of possible ways to decide this issue is not very difficult, so this complication may not present an insurmountable obstacle. But there exist many other problems with the set-based approach to paradox.

An especially difficult problem is caused by the fact that the same set of propositions may be correctly identified with more than one paradox.

For example, the same inconsistent set including both the thesis of geocentrism and that of heliocentrism—the opposed astronomical models that put either the Earth or the Sun at the centre—lends itself to two different paradoxes. Before Copernicus and Galileo finally established heliocentrism as the standard view in the West, thus ending the one-and-a-half millennia-long reign of Ptolemy's geocentric theory, heliocentrism had often been considered a paradigmatic paradox.[2] However, the development of astronomy could have taken a different course: for instance, had the early heliocentrism of Aristarchus become widely accepted in ancient Greece, and been passed on as the standard view to the Romans, then geocentrism—as prominently and ably defended by Aristotle, Ptolemy and others—might have been considered a paradigmatic paradox instead.

One of the motivations behind the set-based approach has been a desire for greater objectivity than is allegedly afforded by the leading argument-based approach inherited from Quine, who famously proposed that 'a paradox is just any conclusion that at first sounds absurd but that has an argument to sustain it' (1962/76, 1).[3]

For example, Quine thought that the barber paradox was best formulated as the true conclusion that there exists no barber who shaves all and only those who do not shave themselves (that is, the negation of p_1); thus, Quine argued that the barber paradox is an instance of what he proposed calling a 'veridical paradox' (an implausible conclusion that is nevertheless true).

From the set-based point of view, the Quinean account of paradox as a kind of implausible conclusion can seem problematic for either of the following two reasons.[4]

First, the Quinean account allows the identification of a paradox on the basis of one's own (subjective) preference for a way of resolving it; in the case of the barber, for example, someone might equally argue that p_4 or p_5, not p_1 (as Quine happened to think), should be negated in order to resolve the paradox, which might in turn suggest a different paradoxical conclusion—say, that p_1

[2] See, for example, the entry 'paradox, *n. (a.)*' in the second (1989) edition of the *Oxford English Dictionary*.
[3] The most prominent Quinean account, beside Quine's, is that by Mark Sainsbury (1988/2009).
[4] Lycan 2010 (617–18) expresses a similar objection.

does not logically imply p_4 and p_5, or simply the negation of p_4 or the negation of p_5—but, contrary to the Quinean account of paradox, such a person would apparently be discussing the same paradox even if there were this kind of disagreement regarding the correct resolution of the paradox and, as a consequence, regarding the choice of conclusion for representing the paradox in the form of an argument.

Second, the Quinean account allows the identification of a paradox on the basis of one's own (subjective) epistemic attitude, specifically on the basis of how plausible one finds individual propositions; however, even if there is agreement regarding the resolution of a given paradox, people may reasonably disagree with one another regarding the plausibility of relevant propositions so that, consequently, they would choose different propositions for *the* paradoxical conclusion, and yet (contrary to the Quinean account) they would appear to be discussing the same paradox.

In response: the second point does not hold universally. The same inconsistent set including both geocentrism and heliocentrism may be agreed, correctly and in accordance with the set-based approach, to constitute a paradox that is resolved by negating geocentrism, but it may nonetheless constitute different paradoxes—namely, either the geocentric or heliocentric paradox—depending on which of the two theses a subject finds more plausible; someone who finds geocentrism more plausible will naturally take the negation of geocentrism to be the paradoxical conclusion (constituting the heliocentric paradox), while someone who finds heliocentrism more plausible will naturally take the negation of heliocentrism to be the paradoxical conclusion (constituting the geocentric paradox). Intuitively, the geocentric paradox is not the same as the heliocentric paradox, even though the two can be represented using the same inconsistent set of propositions. The Quinean account gets this right.

Similarly, as against the first of the two criticisms: would it not be only reasonable to expect the discovery of the correct resolution of a given paradox, if there is one, to potentially coincide with an increase in knowledge of the true nature of the paradox in question (as the Quinean, but not the set-based, account would lead one to expect)? The answer is: yes.

But there exist many other problems with the argument-based approach to paradox. Few philosophical logicians have taken paradoxes to simply be arguments.[5] Just as there are different sets of propositions that may reasonably be taken to represent the same paradox, the same paradox may be given in the form of different arguments with different premises. That is presumably why most people have followed Quine in taking paradoxes to be conclusions of arguments rather than arguments as such, even though it is arguable—as shown by the two criticisms above—that the same paradox may also be given in the form of different conclusions.[6]

Another problem for the Quinean view is that some paradoxes do not seem to come in the form of an argument at all, and therefore not in the form of a conclusion either.

For example, the paradox of the ship of Theseus cannot naturally be cast as an argument with a paradoxical conclusion. Rather, the paradox is normally given in the form of a dilemma. Plutarch tells us that for hundreds of years the people of Athens kept a ship that was said to be the one on which one of their greatest heroes, Theseus, had once triumphantly returned from slaying a dangerous monster on the isle of Crete; however, maintaining the ship over such a long period of time eventually led to most, if not all, of its parts being replaced ('insomuch', Plutarch recounts, 'that this ship became a standing example among the philosophers, for the logical question as to things

[5] Notable exceptions are Mackie 1973 (238) and Priest 1987/2006 (9).
[6] There is further discussion of this latter point below.

that grow; one side holding that the ship remained the same, and the other contending that it was not the same'). Now suppose, in addition, that someone had collected, cleaned up and put back together all of the original parts that had been replaced over time. This gives rise to a dilemma. Which is the ship of Theseus? Is it the 'official' one that was kept and maintained by the people of Athens, or the one that was reassembled from all of the original parts that were gradually replaced over time?[7]

It is possible to cast this paradox into the standard form of an argument, but again which of several candidate paradoxical conclusions would represent, or *be*, the paradox might well be unclear (perhaps that neither ship is the ship of Theseus, or that both are?). In any case, the fact remains that the paradox is not normally—and, apparently, not naturally—presented in this kind of way.

Other well-known paradoxes that are not normally presented in the form of a paradoxical conclusion are the liar paradox, Moore's paradox and Newcomb's problem. Newcomb's problem is normally presented as culminating in a question of choice.[8] Moore's paradox is most frequently given in the form of just one declarative sentence, for example 'It is raining, but I do not believe that it is' (or, perhaps, the more explicit 'It may be true both that it is raining and that I do not believe that it is, yet it is impossible for me to truly assert "It is raining, but I do not believe that it is"'). Similarly, the liar paradox is usually given in the form of just one or two sentences, for example 'This statement is a lie' or 'The following claim is false; the preceding claim is true'; incidentally, Quine presents his own variant of the liar, not as a paradoxical conclusion, but in the form of a mere sentence (namely, '"Yields a falsehood when appended to its own quotation" yields a falsehood when appended to its own quotation').

Other well-known paradoxes, again, are standardly presented in the form of just one interrogative sentence, including the chicken and the egg (which came first?) and the paradox of omnipotence (for example, 'Can God create a stone too heavy for him to lift?').

The defender of the Quinean account of paradox may wish to insist that, even though some paradoxes are not normally presented in the form of an argument or even that of a declarative sentence, all paradoxes can be accurately represented as what, according to the Quinean view, they really are.[9]

Indeed, the hypothesis that any paradox can be *represented* as a conclusion that, in Quine's own words, 'at first sounds absurd but that has an argument to sustain it' proves to be prima facie tenable: it is not difficult to see that representation in the form of a conclusion is in principle possible for all types of paradoxical statements and sentences, including questions and imperatives ('act naturally', 'never say never', 'disobey this order'), and equally for all paradoxes that are naturally presented in the form of subsentential expressions (perhaps the most famous member of this latter category is Russell's paradox of 'the set of all sets that do not contain themselves'; another well-known example is 'the square circle'; other examples are 'alone together', 'bittersweet', 'holy crap', 'living dead').[10]

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[7] See Plutarch [1859], 21. The fictional twist was added by Hobbes.

[8] For the original presentation of Newcomb's problem, see Nozick 1969, 114–15 (which ends: 'Then you make your choice. What do you do?').

[9] The same line of argument may in principle be pursued by defenders of the set-based approach (but they would also face the same sort of problems).

[10] One might think that involvement of non-meaningful expressions—as is arguably the case with the liar paradox, Moore's paradox and others—could make such representation impossible; see, for example,

The metaphysical claim, however, that a paradox essentially *is* what it is thus represented as being—an apparently absurd conclusion that has an argument to sustain it—is false. This becomes especially apparent in the case of non-linguistic paradoxes. But even hybrid cases, part linguistic and part not, constitute obvious counterexamples. Consider, for instance, René Magritte's famous painting that shows a pipe together with the words '*Ceci n'est pas une pipe*' (in English, 'this is not a pipe'). Magritte's painting is called *The Treachery of Images*; an apt title, as it does indeed seem constitutive of the paradox that the viewer is presented with the painted image of a pipe, which supplies the viewer with the intuitive referent of the demonstrative pronoun ('*ceci*', this). If this is true, then mere words cannot replicate this doxastic (belief-involving), or pre-doxastic, situation; therefore, the paradox cannot simply be a conclusion that has an argument to sustain it.

The same holds, even more clearly, for the well-known visual paradoxes popularised by Lionel and Roger Penrose and the artist M. C. Escher, which are constituted by images of so-called impossible objects (for example, the impossible cube, the Penrose triangle and the Penrose stairs). These images produce a special type of visual illusion, in which what appears to the viewer to be the three-dimensional object represented by a two-dimensional image does not merely happen not to be the object represented, but is actually impossible; what is apparently being represented to the subject cannot in fact exist (in particular, the impossible cube is not a cube, and the Penrose triangle is not a triangle). However, even the subject's knowing that this is the case typically does not change the way things visually appear to them (the impossible cube still appears to be a cube, the Penrose triangle still appears to be a triangle).[11] Therefore, mere words cannot replicate the relevant doxastic situation, so the paradox cannot simply be a conclusion that has an argument to sustain it.[12]

In addition, the psychologist Roger Shepard created an auditory paradox that instantiates the same pattern. Shepard showed that a specially devised type of tone, now known as a Shepard tone, can be arranged in a circular manner so as to give the listener the illusion of an infinite musical scale (thus, the paradox is the auditory equivalent of the Penrose stairs). Like in the visual paradoxes, even the subject's knowing about the illusion typically does not change the way things appear to them (the scale still sounds infinite). And, thus, this auditory paradox is another counterexample to the Quinean account.[13]

Sorensen 2003, 352. However, the fact that an expression, or a statement, is non-meaningful can itself be represented in the form of a meaningful statement.

[11] As Penrose and Penrose (1958) note, the illusion is not essentially dependent on two-dimensional images external to the viewer but can equally be produced by a fixed perspective on an actual (but different) three-dimensional object.

[12] Akiyoshi Kitaoka has been perhaps the most prolific producer of visual paradoxes, at least since Vasarely and the op art movement of the twentieth century. The most prominent sort of visual paradox in this connection is images that are not moving but look as if they are; see, for example, Kitaoka's *Rotating Snakes* (2003). As in the case of impossible objects, even the subject's awareness of the illusion typically does not change the way things visually appear to them. In contrast with impossible objects, what looks to the viewer to be the case (say, that the picture is moving) is not impossible but merely happens not to be the case, for it is of course possible to create a variation of one of these images that is actually moving.

[13] See Shepard 1964; a useful sound (and film) recording is Shepard and Zajac 1965. For more paradoxes based on the Shepard tone, see especially Risset 1969 and 1971 and Deutsch 1986, 1992 and 2010. A relatively new type of auditory paradox is the illusion of ambient sound when listening through headphones, which can be created during the production of the recording: as in the case of the Shepard scale, even the subject's awareness of the illusion typically does not change the way things appear to them (although, in contrast, what sounds to the listener to be the case is not impossible but merely happens not to be the case, for it seems possible that the ambient sound that appears to be there could actually be there and be heard). A similar type of tactile paradox is the thermal grill illusion.

As far as the essence of paradox is concerned, then, both the argument-based and the set-based approach prove to be inadequate; both clearly fail to adequately account for all of the cases of paradox mentioned above.

An alternative account has been presented by Roy Sorensen, in his book *A Brief History of the Paradox* (2003). Sorensen writes, 'paradoxes are questions (or in some cases, pseudoquestions) that suspend us between *too many* good answers' (xii). He also writes: 'I take paradoxes to be a species of riddle' (3). And elsewhere in the book he explains the relation between these two claims thus: 'Paradoxes are riddles that overload the audience with good answers … a riddle adopts the form of a question' (349).[14]

Sorensen's account is plausibly interpreted—using the same scheme as above—as the thesis that even though some paradoxes are not normally presented in the form of a question with too many good answers, all paradoxes can be accurately represented in this way as what they really are. But, then, the account will fail in the same way as the Quinean one and for the same sorts of reasons: whilst the hypothesis that any paradox can be represented in the requisite way will probably be true, the accompanying metaphysical claim that a paradox essentially is what it is thus represented as being will be false.

Given Sorensen's expressed intention to offer an account of paradox that is more inclusive than existing argument-based and set-based ones—one that includes, in particular, the Penrose triangle and other visual paradoxes—he would perhaps be inclined to argue that the notion of a question, or that of an answer, should be understood as being sufficiently broad, maybe more like that of a riddle (which would indeed seem broad enough).[15] However, this would result in the rather unhelpful entanglement of the original question, 'what is a paradox?', with one or more of the no-less-difficult questions 'what is a question?', 'what is an answer?' and 'what is a riddle?'

The same sort of problem confronts proponents of argument-based or set-based accounts who try to argue that their respective notions (*argument*, *conclusion*, *proposition*) should be understood as being sufficiently broad.

The account of paradox defended in the present essay, according to which a paradox is something that is beyond belief, avoids this kind of complication. Here the main difficulty is that of seeing how the simplicity of the theory does not lead to excessive inclusivity.

A related difficulty concerns the question of subjectivity: does the account classify as a paradox anything that someone does not believe? That would be an uncharitable interpretation. The phrase 'beyond belief' is intended to imply that a subject has at least made some relevant attempt at believing (more on which below).

The subjectivity of paradox as such—the fact that paradoxicality (the quality of being paradoxical) essentially depends on some suitably related subject with a capacity for belief—is not disputed by anyone. Similarly, no one seriously disputes that paradoxicality comes in degrees; things can be more or less paradoxical than other things or when considered in relation to different subjects or times. For example, the liar paradox is widely considered more paradoxical than that of the barber; Einstein's theory of relativity, and what follows from it, will tend to seem more paradoxical to most non-physicists than most physicists; and heliocentrism is no longer as

[14] Sorensen also refers to his account both as 'my riddle theory of paradoxes' and 'my question-based account' (2003, 358).
[15] He implies as much at one point (2003, 37).

paradoxical for many people as it used to be, or indeed no longer paradoxical at all (except perhaps on occasion, for example whilst watching a sunrise).

Trying to determine a minimum requirement for something to be a ('real') paradox in terms of some specified class of subjects or some degree of paradoxicality is a largely futile undertaking: either no such requirement exists or it is unknowable.

Not restricting in such a way what is counted as a paradox gives a more straightforward, and possibly more precise, theory of what a paradox is, but also possibly one that is extremely inclusive. Extreme inclusivity need not be excessive, however, even if it leads to a surprising number or variety of things that may potentially constitute a paradox.

Rather than solving this problem in a different way, other theories of paradox have instead tended to gloss over it by stipulating what sort of thing a paradox is supposed to be. Naturally, it does not help these theories that all such stipulations appear to fail as accounts of what a paradox is: due to the apparent variety of things that constitute paradoxes—including different kinds of sentence and subsentential expression, pictures and sounds—no such stipulation is likely to be correct.

The number and variety of paradoxes entailed by the unrestricted theory should in fact not be very surprising. Not only do paradigmatic instances of paradox already represent a large variety of things that may constitute a paradox, but standard usage of the words 'paradox' and 'paradoxical' in the English language is actually just as unrestricted as the theory would predict.[16] Besides all kinds of representations and means of representation, we can in fact say of all sorts of different things (actions, apples, events, experiences, persons, reasons, tastes, etc.) that they are a paradox and that they are paradoxical; moreover, we are generally free to relativise these predications to any number of subjects (for example, we can say things like 'I/you/we/they find this man paradoxical'). According to the unrestricted theory, we might in principle be speaking literally and truly each time. If this is correct, then the unrestricted theory will not only be more comprehensive than others, but will be highly adequate.

The account only has to be qualified at the pragmatic level of speech. Although we *can* say literally, and by chance truly, of all sorts of different things that they are paradoxes, we rarely do so even in cases where it would be true. In practice, we tend to reserve the term 'paradox', and to a lesser degree the term 'paradoxical', for certain special cases or occasions. The terms have a mainly honorific use in contemporary English. We tend to only refer to paradoxes as 'paradoxes' (or 'paradoxical') if they are especially deserving of the distinction, because they are paradoxical to some significant degree—rather as we tend to refer to men using the honorific 'Mr' only in some of the cases and on some occasions.

Consequently, the question that philosophers with a general interest in paradox should be asking primarily is not 'what is a paradox?' but 'what makes a paradox philosophically interesting?'

The answer to this question is not a mere function of the degree of paradoxicality. First, what is primarily of interest to philosophers in general is the objects, not the subjects, involved in paradoxes. A paradox that is mainly constituted by a peculiarity in the subject will tend to be of greater interest to psychologists than philosophers. That is why, for example, philosophers in general are not so interested in what might be described as children's paradoxes, like the fact that you cannot reach the end of a rainbow. Generally, philosophers are most interested in cases that

[16] See, for example, the entries 'paradox, *n.* and *adj.*' and 'paradoxical, *adj.*' in the *Oxford English Dictionary*.

tend to be found paradoxical by even the most intelligent subjects. Second, not all objects are of equal interest to philosophers. For example, paradoxes that have arisen in other academic disciplines tend to be of greater interest to practitioners of the relevant discipline than philosophers.[17] Generally, philosophers are most interested in cases that only require a minimal amount of common knowledge to be found paradoxical.

As well as these two principles, the principle of the degree of paradoxicality—the more paradoxical, the better—generally holds. The correct measure of paradoxicality is a difficult question, but there tends to be sufficient agreement regarding the comparative paradoxicality of different cases.[18] The more fundamental question concerns the correct way to analyse paradoxical objects.

To the extent that any general account of paradoxical objects will proceed on the basis of analyses of particular instances, it is likely to favour at least some analyses or resolutions of particular instances over others. But even if it manages to remain largely neutral, the widespread substantive disagreement regarding the correct analysis and resolution of many (if not most) canonical paradoxes gives a clear indication of the difficulty involved in defending any detailed general account of paradoxical objects.

None of the theories criticised above for giving inadequate accounts of the nature of paradox can, as they stand, offer an acceptable general account of paradoxical objects, while the theory that has been proposed as an alternative, according to which a paradox is something that is beyond belief, is lacking in relevant detail. Fortunately, useful detail can be drawn from Quine's original theory, for the core of that theory will be correct if it is interpreted more narrowly than it was intended, as an account not of paradox as such, but of the logical form of paradoxical objects.

On this modified version of Quine's view, then, the logical form of a paradoxical object is a conclusion. The two psychological facts which Quine also mentions—that a paradox is a conclusion that '*at first sounds absurd* but that has an argument *to sustain it*'—are irrelevant in this connection; the mere logical fact that a conclusion has an argument can be left implicit.[19]

The Quinean logical form is more suitable for the analysis of paradoxical objects than those that may be extracted from other theories, mainly because it is more discriminative. Its discriminative power shows the superior accuracy of its logical reduction. By contrast, the set-based account of paradox implies that the logical form of paradoxical objects is an inconsistent set of propositions: not only is this a more complex logical form—and hence a potentially less deep reduction—but its inferior discriminative power was already demonstrated above using the examples of the barber paradox and the geocentric and heliocentric paradoxes.

In the same passage, it was demonstrated that the Quinean account of paradox cannot account for some apparent similarities between cases of different paradoxical conclusions, including cases that intuitively seem to be the same paradox. The set-based approach could account for some of these cases; moreover, it could account for the similarity that appears to unite pairs such as the geocentric and heliocentric paradoxes, insofar as they can be represented as the same set of

[17] See, for instance, the black hole information paradox, the Easterlin paradox, the Einstein–Podolsky–Rosen paradox, the French paradox, *helianthus paradoxus* (paradoxical sunflower), the Hispanic paradox, the lek paradox, Olbers' paradox, the paradox of the plankton, paradoxical embolism, *pseudis paradoxa* (paradoxical frog), *pulsus paradoxus* (paradoxical pulse) and the twin paradox.

[18] See also Paseau 2013.

[19] When interpreted as accounts of the logical form of paradoxical objects, Quine's and Sainsbury's theories are the same.

propositions.[20] On the other hand, the set-based approach cannot account for the similarity that appears to unite versions of the same paradox that employ different arguments, whilst the Quinean account could account for at least some of these cases (namely, those with an identical conclusion). Finally, neither of the two can account for the apparent unity of what are sometimes described as families of paradoxes rather than simply a paradox (for example, the liar paradox). Taking from Quine's original theory only a theory of the logical form of paradoxical objects enables the full use of its discriminative power whilst avoiding some of its explanatory limitations.[21]

Paradoxes can be individuated in extremely fine ways in terms of their subjectivity, including relative to a particular subject at a particular time. But interest in paradoxes usually extends further than this. Indeed, paradoxes are standardly classified on the basis of their paradoxical objects in abstraction from, or idealisation of, their natural subjectivity. Hence, what is called a 'paradox'—including in names of canonical instances—is often really either a type of paradox (relative to an idealised, or otherwise constructed, subject) or a paradoxical object in complete abstraction from all subjectivity.

Identity of paradoxes entails identity of the paradoxical objects, and identity of the paradoxical objects entails identity of the logical form of the paradoxical objects. However, no entailment relation holds in the reverse direction. To reiterate, paradox is a doxastic (belief-involving) phenomenon and, as such, essentially subjective. Not only may what is paradoxical for one subject not be paradoxical for another, but the logical form of a given paradoxical object may itself not be paradoxical for the same subject, because the paradoxical object may be such that what constitutes its paradoxical character gets lost in logical analysis; for example, the paradox may, as in the case of the Penrose triangle or the Shepard scale, be essentially intuitive—that is, constituted by a particular (verbal, visual, etc.) presentation of the paradoxical object.

Thus, identity of the logical form of the paradoxical objects is a necessary, but not a sufficient, condition for the identity of the paradoxical objects. Enquiring into the logical form can help to discriminate paradoxical objects, but it cannot establish identity. In order to do that, one must enquire into the paradoxical object more directly—what is it that is paradoxical?—and a possible outcome of this kind of enquiry will be a doxastic equivalence between the paradoxical object and its logical form, so that the logical form of the paradoxical object is itself paradoxical.

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[20] Pairs like the geocentric and heliocentric paradoxes are not as rare as they might seem. What is basically required is a pair of subjects with contrary beliefs. For example, another instance is the case of someone who, upon hearing the term 'business ethics' for the first time, finds it paradoxical that such a thing (business ethics) should exist—to this person the term seems like an oxymoron—while someone else, upon hearing of the supposed paradox, finds paradoxical the idea that business ethics should *not* exist (given, perhaps, that this person has worked for many years in a field of study that goes by that name). Hence, here again we have two paradoxical conclusions, which the Quinean approach would rightly classify as two different paradoxes; yet the two are also clearly related, which is evident from the fact that they can be represented as the same set of propositions.

[21] Another part of Quine's original theory that will be useful only after modification is his distinction between veridical and falsidical paradoxes. In particular, Quine stipulated that a falsidical paradox is based on a fallacious argument. This needlessly prevents the distinction from applying to all paradoxes, for it excludes those that are not veridical but not based on a fallacious argument either. An improved, simplified distinction runs as follows: a veridical paradox has a true conclusion, a falsidical paradox has a false conclusion. Quine's use of the term 'antinomy', however, should be outright rejected. He says that an antinomy is a paradox that 'produces a self-contradiction by accepted ways of reasoning' (1962/76, 5). But an antinomy is not a kind of paradox at all; it is a kind of contradiction—a contradiction of a law (see also Kant 1981/7, A407/B434). Thus, the kind of paradox that Quine rightly singles out as of special importance might be better described as one that involves an antinomy.

Consider once more the possible disagreement regarding the paradox of the barber who shaves all and only those who do not shave themselves, where one subject takes the paradoxical conclusion, out of the set of propositions p_1–p_5 (as above), to be the negation of p_1 ('There exists a barber who shaves *x* if, and only if, *x* does not shave *x*'), and the other takes it to be the negation of the conjunction of p_4 ('If the barber shaves the barber, then the barber does not shave the barber') and p_5 ('If the barber does not shave the barber, then the barber shaves the barber'). Suppose each subject has thus correctly determined both what it is that is paradoxical for them and its logical form; hence, what it is that is paradoxical in each case differs in logical form. It follows that there are, independently of all subjectivity, two different paradoxical objects and, hence, two different paradoxes.

It is tempting to say that these are really versions of the same paradox. But this temptation should be resisted. For it would result in a false account of the disagreement, which concerns the true nature of the (type of) paradox. The term 'version' should instead be reserved for cases of different paradoxical objects of the same logical form (so, for example, different arguments to the same paradoxical conclusion).

Of course, the disagreement arises only on the assumption that these are two analyses of what on the surface appears, and in a sense is agreed, to be the same paradox. This can be explained by the fact that there exists a description—like a common multiple—of the different paradoxical objects under which they are apparently identical (for instance, 'the barber paradox' or 'the paradox of the barber who shaves all and only those who do not shave themselves').

What unites the geocentric and heliocentric paradoxes can be explained in the same way. For example, 'the spatial relation between the Earth and the Sun' is a description of the different paradoxical objects under which they are apparently identical.

Moreover, the unity of what are sometimes called families of paradoxes can be explained in this way, too. Take for example sorites paradoxes, such as 'One grain of rice does not make a heap; if one grain does not make a heap, then neither do two; if two do not, then three do not; and so on; thus, one million grains of rice do not make a heap' or 'If a man with only one hair on his head is bald, then all men are bald'.[22] What unites different sorites paradoxes that have different paradoxical objects and different logical forms is that there exists a description of the different paradoxical objects under which they are apparently identical (for example, 'vagueness'). Similarly, what unites different liar paradoxes that have different paradoxical objects and different logical forms is that there exists a description of the different paradoxical objects under which they are apparently identical (for example, 'truth').

The question of what holds a given, or putative, family of paradoxes together is a difficult one. For example, 'truth' and 'vagueness' will almost definitely not be correct, or at any rate not be the only correct, answers for liar and sorites paradoxes, respectively. Any account of what holds a family of paradoxes together will depend on analyses of the paradoxical objects of the different candidate paradoxes. When analysing a given paradoxical object, one is looking for the final analysis of it that gives something doxastically equivalent (and is hence itself paradoxical), which might also be called 'the paradoxical element'. When comparing different paradoxical objects, on the other hand, one is looking for the first (or most adequate) description—like the least common multiple—of the different paradoxical objects under which they will be apparently identical (what might be called 'the paradoxical product'). Recent work on liar paradoxes shows the epistemic potential of pursuing

[22] The word 'sorites' comes from the ancient Greek '*soros*', which means heap.

both these analytic and synthetic types of enquiry for the study of particular paradoxes (for example, particular liar paradoxes).[23]

It has also been said of the paradox of omnipotence that it is really a family of paradoxes; and, in fact, the example that was given above is more commonly known as the paradox of the stone. However, the paradox of omnipotence is probably not as large a family as it might seem, and perhaps not a family at all, because most if not all things that have gone by that name—including the paradox of the stone—would seem to be correctly analysed as versions of the same paradox, sharing the same paradoxical conclusion (that there can be no omnipotent being).

To sum up, a paradox is something that is beyond belief. However, we do not normally call everything that is a paradox by that name; rather, we tend to confer the predicate like a title, based on the degree of paradoxicality. It nonetheless follows from the definition of paradox as something beyond belief that anything whatsoever is potentially a paradox. This is not a bad thing. Russell famously declared paradox to be the end, or aim, of philosophy: 'The point of philosophy is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it' (1918, 514). In other words, a good philosopher is someone who can make the familiar, or trivial, seem strange. Of course, this is equally to declare paradox to be the start of philosophy—the root, as it were, out of which further discourse grows. That is the fundamental epistemic value of finding things paradoxical. The unrestricted theory is a reminder that paradoxes may in principle be found everywhere and in everything.

Finally, there remains the issue that the theory may seem to imply that, by logical necessity, most things actually are paradoxes, because most things are *logically* beyond belief (apples, for instance, are not the kind of thing that can be believed). But this is to overlook the subjectivity of paradox and, hence, to misconstrue the meaning of 'beyond belief': given the subjectivity of paradox, it should be understood in this connection as entailing that a subject has at least made some relevant attempt at believing. For example, suppose that someone has never come across a red-fleshed apple before. Upon their first encounter, they are unsure what to think of it; they thoroughly examine the strange fruit, and find it paradoxical; it looks like a normal apple on the outside, but not inside, and it smells and tastes like sweet berries. The apple is beyond belief, a paradox, but precisely what the issue is may not yet be known: is the issue whether this even is an apple or, perhaps, whether the fruit really is the way it seems (looks, tastes, etc.)? Thus, the most accurate description of the paradoxical object, at this time, is something like 'that apple'. Identifying the paradoxical element of a given paradox is difficult. And the world is full of strange fruit.

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[23] For a useful and relevant overview of this work, see Beall, Glanzberg and Ripley 2019.

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